MA103 - Class 10

THM Let $I, J \subseteq \mathbb{R}$ be intervals and $I \xrightarrow{f} J \xrightarrow{g} \mathbb{R}$ be cont. functions. Then $g \circ f$ is continuous.

EXERCISE 3.17 a) if $g \circ f$ is continuous at a, then f is continuous at a AND g is continuous at $f(a)$

b) if $g \circ f$ is continuous at a, then f is continuous at a OR g is continuous at $f(a)$

$f(x) = \mathbb{1}_{\{0\}}$ $\qquad$ $g(x) = -\mathbb{1}_{\{1\}}$

① Is it given by the thm?
② If not, what do we want from our counterexample?

Ⓐ f, g NOT continuous
Ⓑ $g \circ f$ continuous

constant function

$\to$ what is the easiest example?
- $\mathbb{1}_A$
- $f(x) = \begin{cases} \ldots \\ \ldots \end{cases}$

OTHER EXAMPLES?

c) if $g \circ f$ is not continuous at a, then f is not continuous at a OR g is not continuous at $f(a)$

THM: f cont $\wedge$ g cont $\Rightarrow$ $g \circ f$ cont

equivalently, $g \circ f$ not cont $\Rightarrow$ f not cont $\vee$ g not cont

TRUE

d) if $g \circ f$ is not continuous at a, then f is not continuous at a AND g is not continuous at $f(a)$

want: • $g \circ f$ not continuous
• $\neg$(f not cont and g not cont)

$f(x) := x$ $\qquad$ $g(x) := \mathbb{1}_{\{0\}}$.

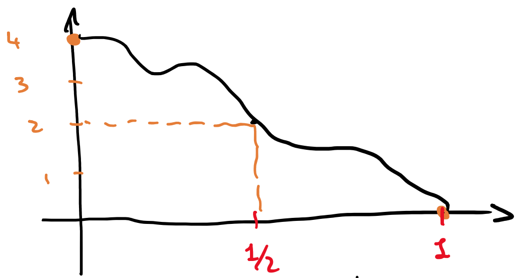
EXER 3.18 Let $f(x) := \begin{cases} x \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$. Is f cont. at 0?

THM (INTERMEDIATE VALUE THM) Let $f: [a,b] \to \mathbb{R}$ be continuous.
$\forall y \in [f(a), f(b)]$ $\exists c \in [a,b]$ s.t. $f(c) = y$

SUPER USEFUL ① Computational uses / show that p... has a root in....

② Fixed point theorem

③ Let $f: [0,1] \to \mathbb{R}$ continuous. $\exists c \in [0,1]$ s.t.
$f(c) - f(1) = (f(0) - f(1))c$



Let $g(x) := f(x) - f(1) - (f(0) - f(1))c$
We have $f(c) - f(1) = (f(0) - f(1))c$ iff $g(c) = 0$.

④ A man climbs a mountain ....

THM (EXTREME VALUE THM) Let $f: [a,b] \to \mathbb{R}$ be a continuous function.
Then ① $f([a,b])$ is bounded
② $\sup(f([a,b]))$ is attained

THM (EXTREME VALUE THM) Let $f: [a,b] \to \mathbb{R}$ be a continuous function.
Then $\exists c, d \in \mathbb{R}$ s.t. $f([a,b]) = [c,d]$.

EXER 3.31 A continuous periodic function is bounded.

$\rightsquigarrow f(\mathbb{R}) = f([0,T])$ which is bounded by extreme value thm.

EXER 3.32 Let $f : [a,b] \to \mathbb{R}$ s.t. $\forall x \in [a,b]$, $f(x) > 0$.
Is it true that $\exists \delta > 0$ s.t. $\forall x \in [a,b]$, $f(x) > \delta$??