MA210 - Class 5

Q Prove that if two graphs are isomorphic, they have the same degree sequence.

$\rightsquigarrow$ Let $G = (V, E)$ and $G' = (V', E')$ be two graphs, and let $\varphi : V \to V'$ be a graph isomorphism.

CLAIM $\forall v \in V, \quad \varphi(N(v)) = N(\varphi(v))$.

Indeed, we have

$$
\begin{aligned}
y \in \varphi(N(v)) &\iff \exists x \in N(v) \text{ s.t. } y = \varphi(x) \\
&\iff \exists x \in V;\ xv \in E \text{ and } y = \varphi(x) \\
&\iff \exists x \in V;\ \varphi(x)\varphi(v) \in E' \text{ and } y = \varphi(x) \\
&\iff y\varphi(v) \in E' \text{ and } \exists x \text{ s.t. } \varphi(x) = y \\
&\iff y\varphi(v) \in E' \iff y \in N(\varphi(v))
\end{aligned}
$$

This is enough to conclude, since φ is a bijection and therefore it preserves cardinalities.

Q Find all non-isom. 3-regular graphs on 6 vertices

$\rightsquigarrow$ METHOD A: As done by most

METHOD B:

CLAIM G and G' are isom. iff $\overline{G}$ and $\overline{G'}$ are.

CLAIM If G is k-regular on n vertices, $\overline{G}$ is $n-k-1$ regular.

Given these two claims (which require a 2-lines proof). We can ask the question: how many 2-regular graphs are there on 6 vertices?

CLAIM A 2-regular graph is the disjoint union of cycles.

Given all these claims the result follows immediately.

Q Let G be a graph s.t. $\forall v \in V$, $d(v) \geq k$ for some fixed $k \in \mathbb{N}^+$).

a) Show that G contains a $(k+1)$-vertex path

b) ——— " ——— a cycle on AT LEAST $k+1$ vertices.

Sol a) ① TAKE THE LONGEST PATH P

② IT CANNOT BE EXTENDED

③ So $N(x) \subseteq P$ for x an endpoint.

b) We can get one from point a).

Q Let G be a graph with $|V(G)| \geq 2$. Then there exist $v, w \in V(G)$ s.t. $d(v) = d(w)$.

Easy if done the right way.

Impossible using the wrong approach.