Partition universality

Domenico Mergoni Cecchelli

with P. Allen, J. Böttcher

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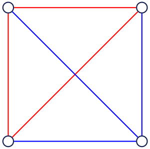

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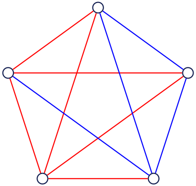

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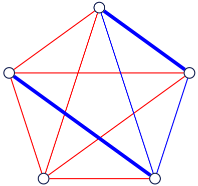

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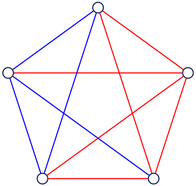

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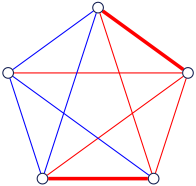

size-Ramsey number

Towards size-Ramsey

Are there smaller $2K_2$-Ramsey graphs?

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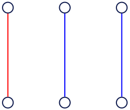

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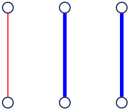

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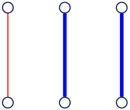
$3K_2 \rightarrow_2 2K_2$



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$\hat{r}_2(2K_2)$

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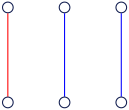
$\hat{r}_2(2K_2) = 3$



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The r-size-Ramsey number of F is the minimum $e(\Gamma)$ such that $\Gamma \to_r F$.

- *Rem:* We have $\hat{r}_r(F) \leq \binom{r_r(F)}{2}$

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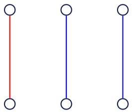

Bounds

Thm. (Chvátal, Rödl, Szemerédi and Trotter, 1983)

If F is a graph on n vertices with $\Delta(F) \leq \Delta$, then:

$$e(F) \leq \hat{r}_r(F) = O(n^2).$$

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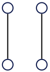

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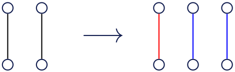

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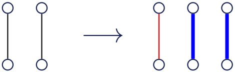

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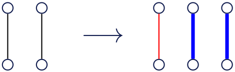

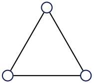

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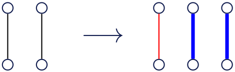

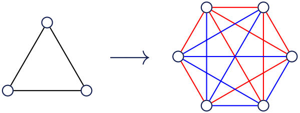

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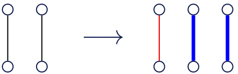

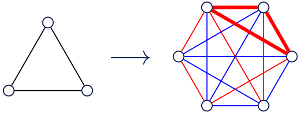

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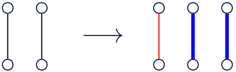

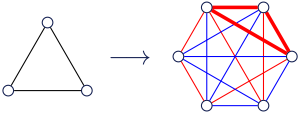

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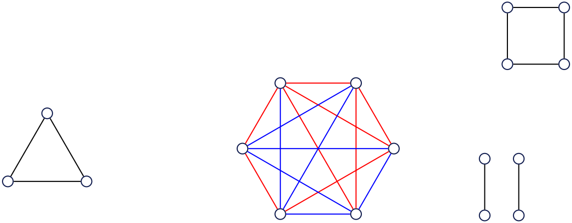

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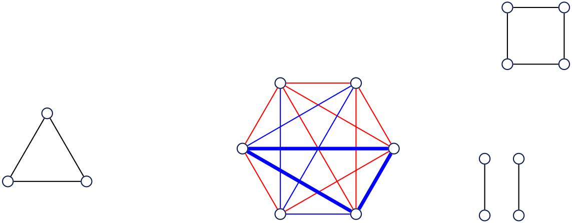

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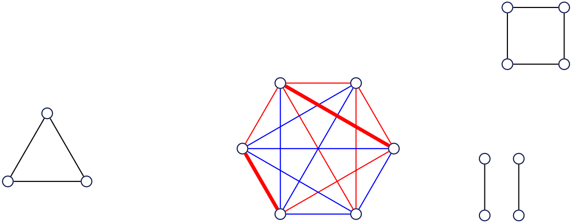

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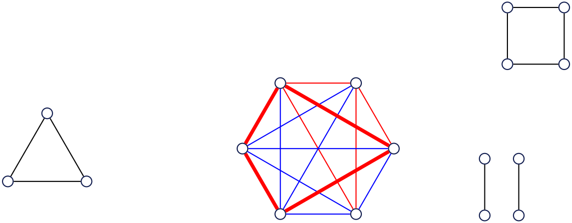

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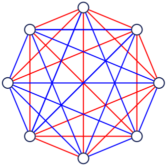

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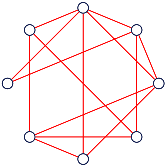

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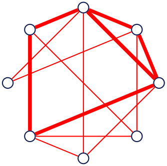

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$G(Cn, (\frac{Cn}{\log Cn})^{-\frac{1}{\Delta}})$, a.a.s. is r-partition universal for $\mathcal{G}(\Delta, n)$.

Summary

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While studying upper bounds for $\hat{r}_r(\mathcal{G}(\Delta, n))$ one realises that the p for which we can prove $G(Cn, p)$ is r size-Ramsey for $\mathcal{G}(\Delta, n)$ are the same for which $G(Cn, p)$ is r-partition universal for $\mathcal{G}(\Delta, n)$.

Our results

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A.a.s. $G(Cn, (\frac{Cn}{\log Cn})^{-\frac{1}{\Delta}})$ is r-partition universal for $\mathcal{G}(\Delta, n)$.

Our results

GOAL: Study partition universality properties of random graphs.

Thm. (Allen, Böttcher, 2022)

A.a.s. $G(Cn, (Cn)^{\mu - \frac{1}{\Delta - 1}})$ is r-partition universal for $\mathcal{G}(\Delta, n)$.

Our results

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Thm. (Allen, Böttcher, 2022)

A.a.s. $G(Cn, (Cn)^{\mu - \frac{1}{\Delta-1}})$ is r-partition universal for $\mathcal{G}(\Delta, n)$.

Rem: Worst bounds for $\Delta = 3$.

Our results

GOAL: Study partition universality properties of random graphs.

Thm. (Allen, Böttcher, 2022)

A.a.s. $G(Cn, (Cn)^{\mu - \frac{1}{\Delta - 1}})$ is r-partition universal for $\mathcal{G}(\Delta, n)$.

Coro. (Allen, Böttcher, 2022)

For any $F \in \mathcal{G}(\Delta, n)$,

$$\hat{r}_r(F) = O(n^{2+\mu-\frac{1}{\Delta-1}}).$$

Our results

GOAL: Study partition universality properties of random graphs.

Lemma/Thm. (Allen, Böttcher, 2022)

A.a.s. $G(Cn, (Cn)^{\mu - \frac{1}{D}})$ is r-partition universal for $\mathcal{G}(D, \Delta, n)$.

Our results

GOAL: Study partition universality properties of random graphs.

Lemma/Thm. (Allen, Böttcher, 2022)

A.a.s. $G(Cn, (Cn)^{\mu-\frac{1}{D}})$ is r-partition universal for $\mathcal{G}(D, \Delta, n)$.

Rem: - By first moment method we cannot take $p = o((Cn)^{-\frac{1}{D}})$.
- Any r-partition universal Γ for $\mathcal{G}(D, 2D+1, n)$ has $e(\Gamma) \geq \frac{1}{100} n^{2-\frac{1}{D}}$.

Our results

GOAL: Study partition universality properties of random graphs.

Lemma/Thm. (Allen, Böttcher, 2022)

A.a.s. $G(Cn, (Cn)^{\mu - \frac{1}{D}})$ is r-partition universal for $\mathcal{G}(D, \Delta, n)$.

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Thm. (Allen, Böttcher, M.C., 2023+)

$G^{(k)}(Cn, (Cn)^{\mu - \frac{1}{D}})$ is a.a.s. r-partition universal for $\mathcal{G}^{(k)}(D, \Delta, n)$.

Our results

GOAL: Study partition universality properties of random graphs.

Lemma/Thm. (Allen, Böttcher, 2022)

A.a.s. $G(Cn,(Cn)^{\mu-\frac{1}{D}})$ is r-partition universal for $\mathcal{G}(D,\Delta,n)$.

Rem: - By first moment method we cannot take $p=o((Cn)^{-\frac{1}{D}})$.
- Any r-partition universal Γ for $\mathcal{G}(D,2D+1,n)$ has $e(\Gamma)\geq\frac{1}{100}n^{2-\frac{1}{D}}$.

Thm. (Allen, Böttcher, M.C., 2023+)

$G^{(k)}(Cn,(Cn)^{\mu-\frac{1}{D}})$ is a.a.s. r-partition universal for $\mathcal{G}^{(k)}(D,\Delta,n)$.

Thm. (Cooley, Fountoulakis, Kühn, Osthus, 2009)

$G^{(k)}(Cn,1)$ is a.a.s. r-partition univ. for $\mathcal{G}^{(k)}(\Delta,n)$.

Our results

<u>GOAL</u>: Study partition universality properties of random graphs.

Lemma/Thm. (Allen, Böttcher, 2022)

A.a.s. $G(Cn, (Cn)^{\mu - \frac{1}{D}})$ is r-partition universal for $\mathcal{G}(D, \Delta, n)$.

Rem: - By first moment method we cannot take $p = o((Cn)^{-\frac{1}{D}})$.
- Any r-partition universal Γ for $\mathcal{G}(D, 2D+1, n)$ has $e(\Gamma) \geq \frac{1}{100} n^{2-\frac{1}{D}}$.

Thm. (Allen, Böttcher, M.C., 2023+)

$G^{(k)}(Cn, (Cn)^{\mu - \frac{1}{D}})$ is a.a.s. r-partition universal for $\mathcal{G}^{(k)}(D, \Delta, n)$.

Thm. (Allen, Böttcher, Hng, Skokan, Davies, 2021)

$G^{(k)}(Cn, (Cn)^{-\epsilon})$ is a.a.s. r-partition univ. for $\mathcal{G}^{(k)}(\Delta, n)$.

Idea of the proof

Want

A.a.s. $G^{(2)}(Cn, (Cn)^{\mu-\frac{1}{D}})$ is r-partition universal for $\mathcal{G}^{(2)}(D, \Delta, n)$.

Idea of the proof

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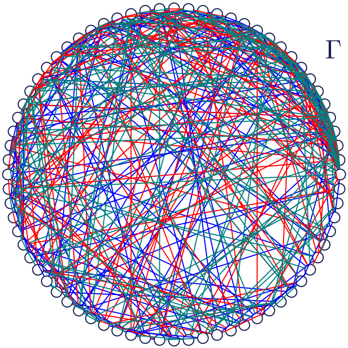
A.a.s. $G^{(2)}(Cn, (Cn)^{\mu-\frac{1}{D}})$ is r-partition universal for $\mathcal{G}^{(2)}(D, \Delta, n)$.



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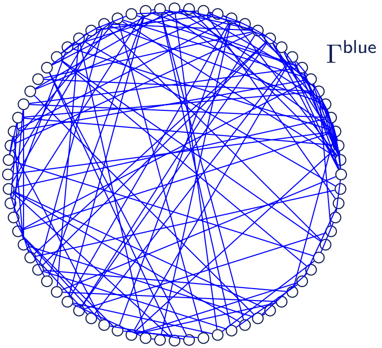

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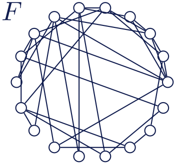

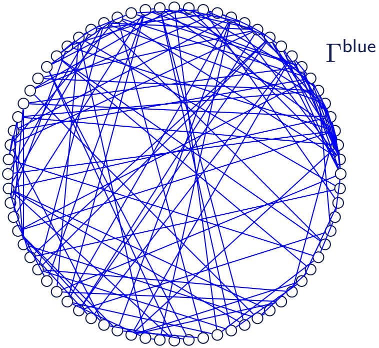

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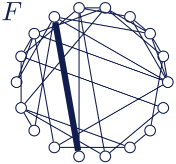

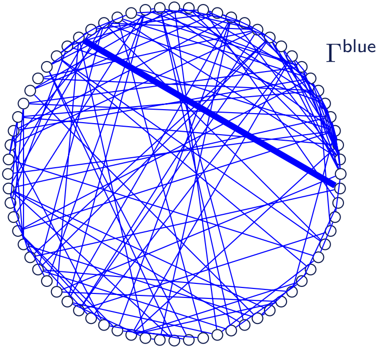

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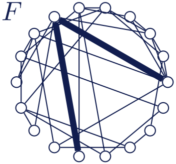
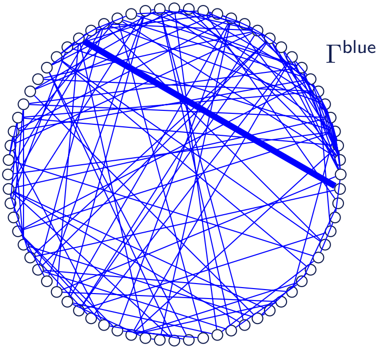
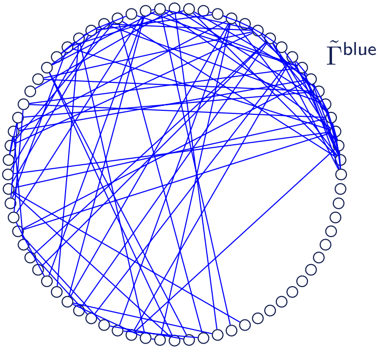




Idea of the proof

Want

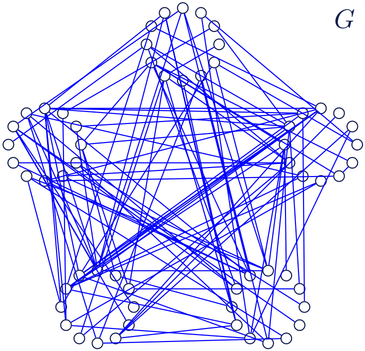
A.a.s. $G^{(2)}(Cn,(Cn)^{\mu-\frac{1}{D}})$ is r-partition universal for $\mathcal{G}^{(2)}(D,\Delta,n)$.



Idea of the proof

Want

A.a.s. $G^{(k)}(Cn,(Cn)^{\mu-\frac{1}{D}})$ is r-partition universal for $\mathcal{G}^{(k)}(D,\Delta,n)$.



Idea of the proof

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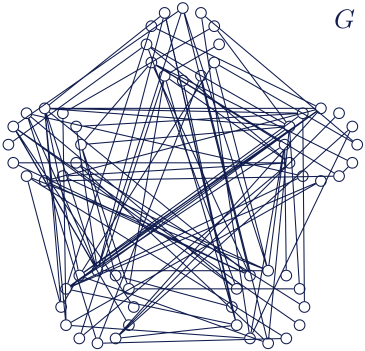

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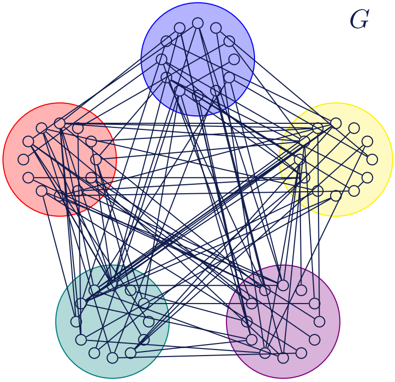

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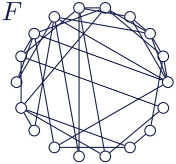

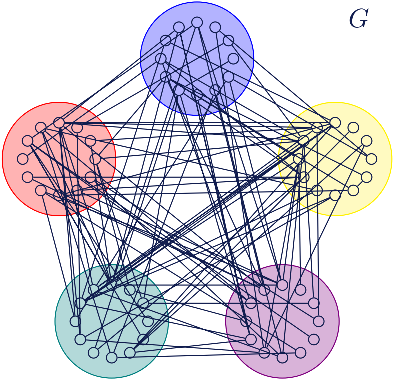

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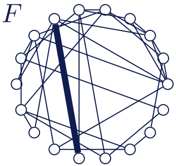

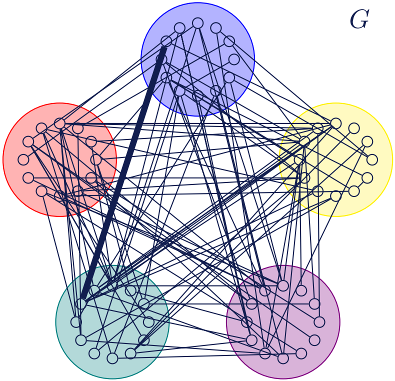

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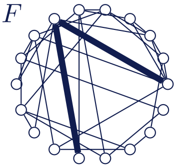

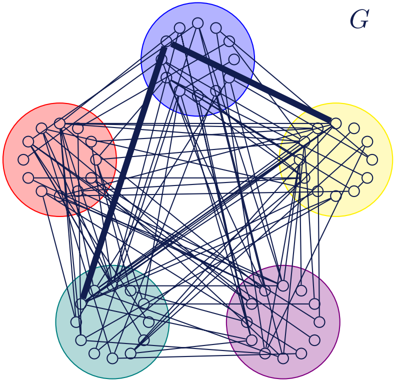

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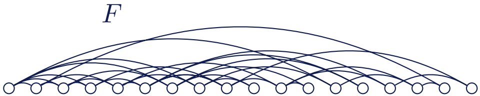

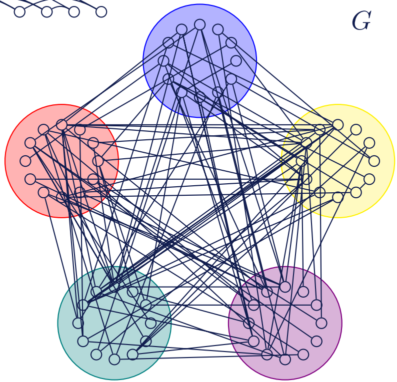

Idea of the proof

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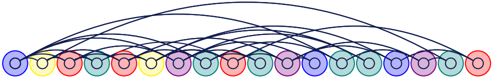
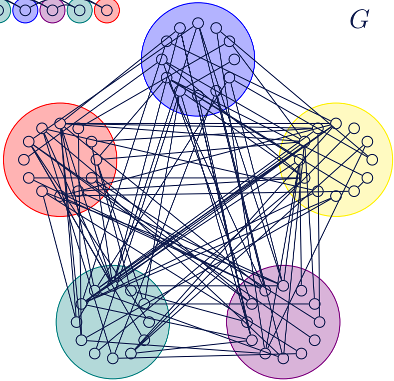


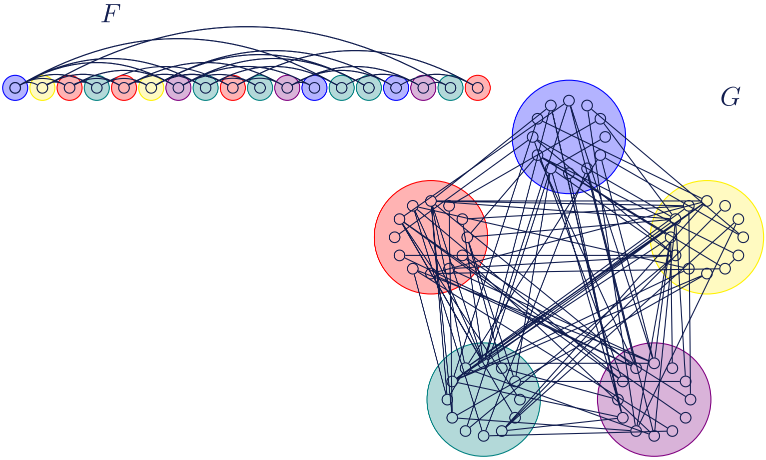


In F, vertices with the same colour are distant.

Idea of the proof

Goal

Find $\psi_1, \ldots, \psi_{v(F)}$ growing seq. of partial homom. from F to G.



Idea of the proof

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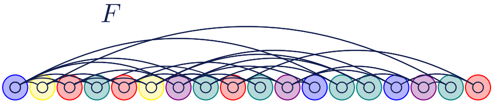
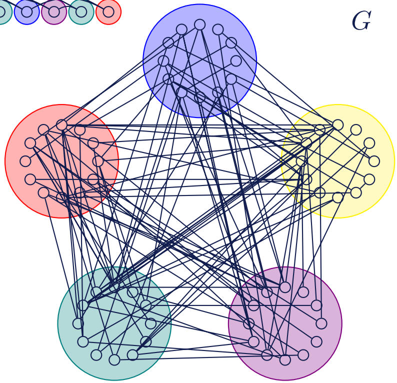


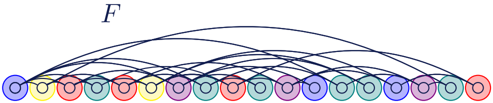


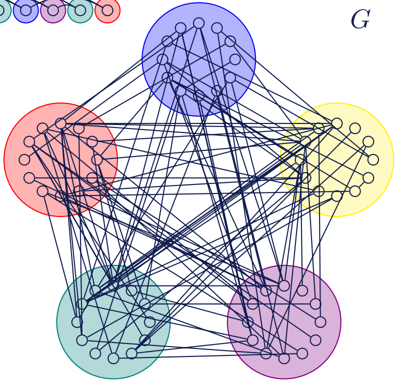
At each step, the number of viable embeddings of each of the following vertices should not decrease too much.

Idea of the proof

Goal

Find $\psi_1, \ldots, \psi_{v(F)}$ growing seq. of partial homom. from F to G.

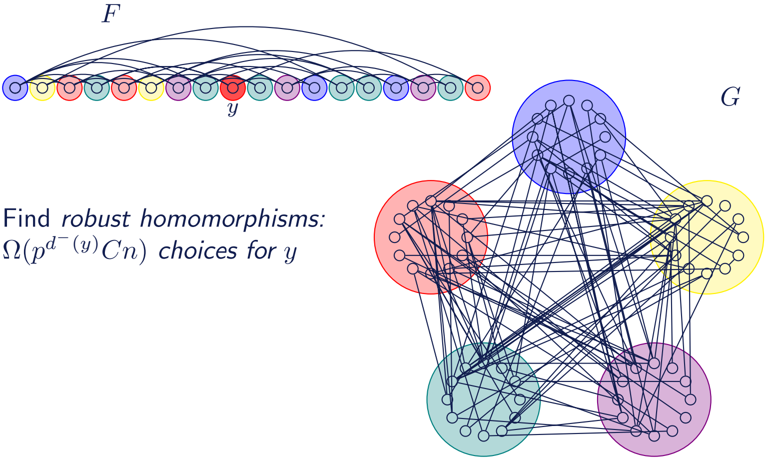




Find *robust homomorphisms*

Goal

Find $\psi_1, \ldots, \psi_{v(F)}$ growing seq. of partial homom. from F to G.

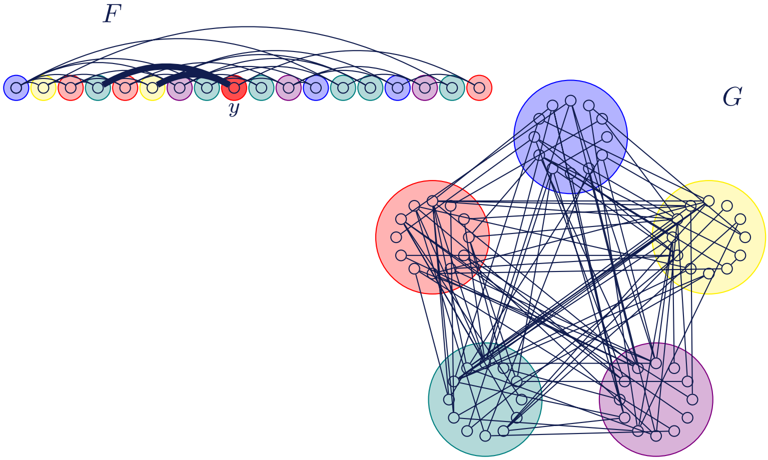


Find *robust homomorphisms:* $\Omega(p^{d^-(y)}Cn)$ *choices for* y

Idea of the proof

Goal

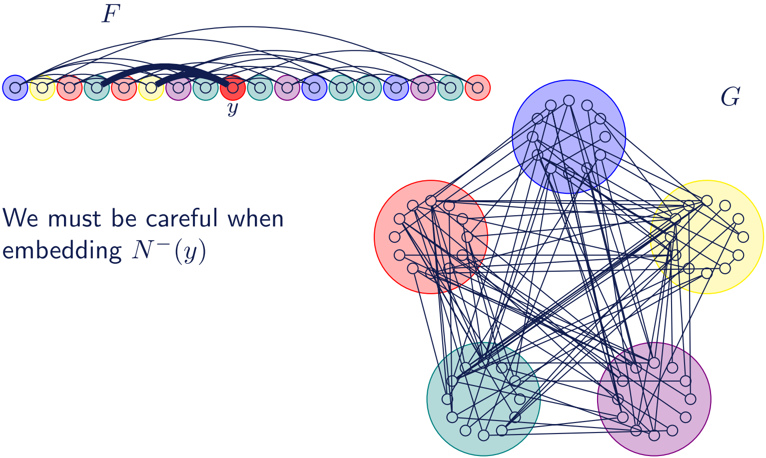
Find $\psi_1, \dots, \psi_{v(F)}$ growing seq. of partial homom. from F to G.



Idea of the proof

Goal

Find $\psi_1, \ldots, \psi_{v(F)}$ growing seq. of partial homom. from F to G.

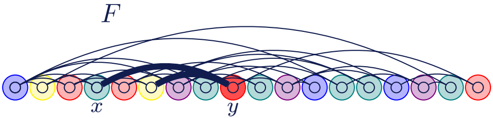


We must be careful when embedding $N^-(y)$

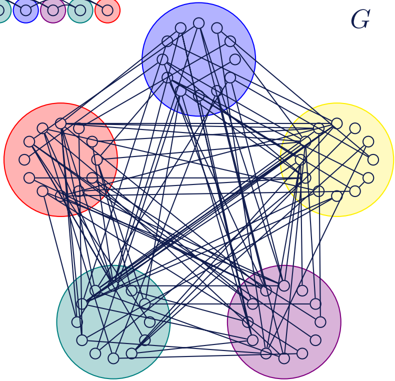
Idea of the proof

Goal

Find $\psi_1, \ldots, \psi_{v(F)}$ growing seq. of partial homom. from F to G.



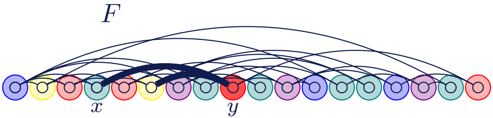
When embedding x we need to consider y.



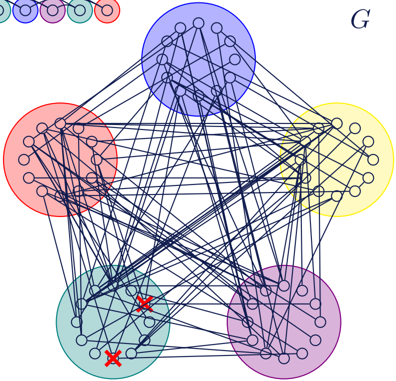
Idea of the proof

Goal

Find $\psi_1, \ldots, \psi_{v(F)}$ growing seq. of partial homom. from F to G.



We won't embed x to *dangerous* vertices, but there are very few of them.



Idea of the proof

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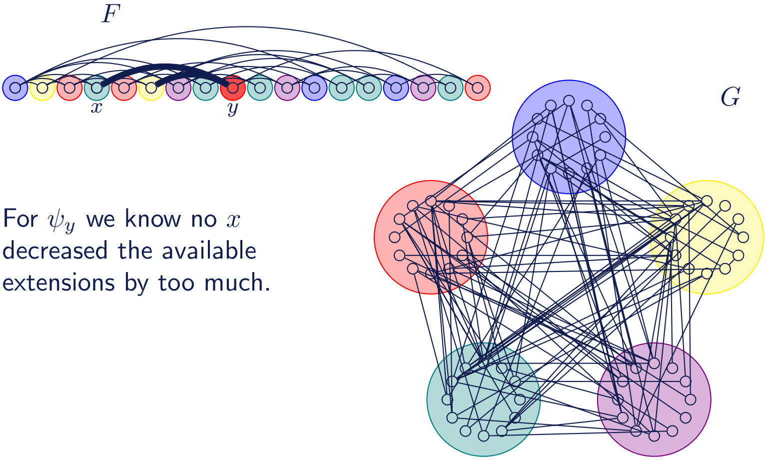

For ψ_y we know no x decreased the available extensions by too much.


Thank
You!!